Department of Commerce and Labor

BULLETIN
OF THE
BUREAU OF STANDARDS

Volume 4
1907-8

DEPARTMENT OF COMMERCE AND LABOR

BULLETIN

OF THE

BUREAU OF STANDARDS

S. W. STRATTON, Director

VOLUME 4

(Nos. 1, 2, 3, 4)

1907-8

WASHINGTON
GOVERNMENT PRINTING OFFICE
1908

EFFECT OF WAVE FORM UPON THE IRON LOSSES IN TRANSFORMERS.

By Morton G. Lloyd.

It has long been known that the iron loss in a static transformer is dependent upon the form of the wave of electrical pressure which is applied to its primary winding, since the form of this wave, with the constants of the transformer, determines the wave of magnetic flux produced in the iron core. In 1895 Dr. Roessler[1] read a paper before the Verband Deutscher Electrotechniker in which he presented the results of experiments, using two generators whose curves of electromotive force were quite different. With the same emf. applied to the transformer, the iron losses were decidedly less with a peaked emf. wave, corresponding to a flat wave of magnetic flux. He also pointed out that when the form of wave is not a sine curve, it is necessary to modify the formula of Steinmetz for iron losses by introducing the form factor of the emf. wave.

As the Bureau of Standards possesses facilities for supplying emf. waves of any desired form, the present investigation was undertaken, at the suggestion of Dr. E. B. Rosa, for the purpose of shedding additional light upon this question. The problem is first considered from the theoretical standpoint, and the results of experiments are given in which various wave forms were used with commercial types of transformers. The wave forms were obtained by using a series of generators[2] giving approximately sine waves of frequencies 60, 180, 300, 420, 540, 660, 780, and 900 cycles per second when running at normal speed. The phase relations of the several generators are adjustable, and as they are mounted upon a single shaft, remain

[1] G. Roessler. Electrician, **36**, p. 124; 1895.

[2] A full description of this set of generators will be given in a subsequent issue of this Bulletin.

invariable during an experiment. By connecting these generators in series and exciting each to the proper voltage, any desired wave form which involves only the odd harmonics up to the fifteenth may be procured.

1. THEORETICAL.

Let W = power expended in the core of a transformer.

W_H = power expended in hysteresis.

W_E = power expended in eddy currents.

B = maximum flux density in the iron, the flux being assumed uniform.

η and ζ be constants depending upon the iron.

q = ratio of hysteresis to total iron loss.

$n = \frac{1}{T}$ = frequency of applied emf.

f = form factor of induced emf.

e = instantaneous value of induced emf.

E = effective value of induced emf.

E = average value of induced emf. for a half period.

A = amplitude of fundamental component of induced emf.

h = ratio of amplitude of harmonic to amplitude of fundamental.

m = order of harmonic.

θ = phase angle of harmonic.

t = time since fundamental passed through value zero in positive direction.

t_1 = time at which $e=0$.

$\phi = \frac{2\pi t}{T}$

$\phi_1 = \frac{2\pi t_1}{T}$

The subscript zero applies to the case of a sine wave.

DEFINITIONS.

1. The effective value of a periodically varying quantity is the square root of the average value of the square of that quantity.

2. The form factor of a periodically varying quantity is the ratio of its effective value to the algebraic average value during the half period from t_1 to $t_1 + \frac{T}{2}$.

We assume the iron losses in the transformer to be represented by the modified form of the Steinmetz formula,

$$W = W_H + W_E$$

$$= \eta n B^{1.6} + \zeta f^2 n^2 B^2$$

The induced voltage in the primary winding of a transformer differs from the applied voltage by the amount of the ohmic drop, or difference of potential due to resistance. As the ohmic drop is small, and as consideration of it introduces difficulties, we shall consider only the induced voltage. Commercial voltmeters measure the effective value of the voltage. We assume consequently that with different wave forms the effective value of the induced voltage is the quantity kept constant. This approximates closely to practical conditions.

Since the induced voltage depends upon the rate of change of the magnetic flux, its average value at a definite frequency is proportional to B, and its effective value to fB. If the effective value be kept constant, the term in the above formula representing eddy currents will be constant at any definite frequency. The only change in W will arise from an alteration in B, which must accompany any alteration in f.

$$W_0 = W_{H_0} + W_E = \frac{W_{H_0}}{q}$$

$$\frac{W - W_0}{W_0} = \frac{W_H - W_{H_0}}{W_0} = q\left(\frac{W_H}{W_{H_0}} - 1\right) = q\left[\left(\frac{B}{B_0}\right)^{1.6} - 1\right]$$

$$= q\left[\left(1 + \frac{B - B_0}{B_0}\right)^{1.6} - 1\right]$$

If $B - B_0$ be small in comparison with B_0, this may be written

$$1.6\, q \frac{B - B_0}{B_0}$$

If B and q be known, the change in power may be calculated.

q may be determined approximately by runs at two frequencies, using sine waves and voltages proportional to the frequencies. One frequency n_1 and voltage should be those used with the varying

wave forms. Under these conditions the total flux remains constant and B is constant. Hence

$$W_1 = \eta\, n_1 B^{1.6} + \zeta\, n_1^2 f^2 B^2$$

$$W_2 = \eta\, n_2 B^{1.6} + \zeta\, n_2^2 f^2 B^2$$

and by subtraction, after dividing by frequency,

$$\zeta f^2 B^2 = \frac{\dfrac{W_1}{n_1} - \dfrac{W_2}{n_2}}{n_1 - n_2}$$

from which

$$q = \frac{n_1 \dfrac{W_2}{n_2} - n_2 \dfrac{W_1}{n_1}}{(n_1 - n_2)\dfrac{W_1}{n_1}}$$

It remains to determine B for various forms of the emf. wave. B is proportional to the average value of the induced voltage during the time in which the flux changes from a positive maximum to a negative maximum. We assume the positive and negative halves of the wave to be alike. This is true of the emf. wave of any well-designed and well-constructed generator, which will therefore contain only the odd harmonics of its fundamental.

Any such emf. may be represented by the equation

$$e = A \sin \frac{2\pi t}{T} + A h_3 \sin\left(\frac{6\pi t}{T} + \theta_3\right)$$
$$+ A h_5 \sin\left(\frac{10\pi t}{T} + \theta_5\right) + A h_7 \sin\left(\frac{14\pi t}{T} + \theta_7\right)$$
$$+ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

$$E^2 = \frac{2}{T}\int_0^{\frac{T}{2}} e^2 dt = \frac{A^2}{2}\left(1 + h_3^2 + h_5^2 + h_7^2 + \quad . \quad . \quad . \quad . \right)$$

$$E = \frac{A}{\sqrt{2}}\sqrt{1 + h_3^2 + h_5^2 + h_7^2 + \quad . \quad . \quad . \quad . \quad . \quad .}$$

In our case E is to remain constant. For the sine wave $E=\frac{A_0}{\sqrt{2}}$

Hence $$\frac{A}{A_0}=\frac{1}{\sqrt{1+h_3^2+h_5^2+\ldots}}$$

$$\mathsf{E}=\frac{2}{T}\int_{t_1}^{t_1+\frac{T}{2}} e\,dt=\frac{1}{\pi}\int_{\varphi_1}^{\varphi_1+\pi} e\,d\phi$$

$$=\frac{1}{\pi}\int_{\varphi_1}^{\varphi_1+\pi} A\sin\phi\,d\phi+\frac{1}{\pi}\int_{\varphi_1}^{\varphi_1+\pi} Ah_3\sin(3\phi+\theta_3)\,d\phi$$

$$+\frac{1}{\pi}\int_{\varphi_1}^{\varphi_1+\pi} Ah_5\sin(5\phi+\theta_5)\,d\phi+\ldots\ldots$$

$$=\frac{2A}{\pi}\left[\cos\phi_1+\frac{1}{3}h_3\cos(3\phi_1+\theta_3)+\frac{1}{5}h_5\cos(5\phi_1+\theta_5)+\ldots\right]$$

$$\mathsf{E}_0=\frac{2A_0}{\pi}$$

If the harmonics are in phase with the fundamental

$$0=\theta_3=\theta_5=\theta_7=\text{ etc. and }\phi_1=0.$$

$$\mathsf{E}=\frac{2A}{\pi}\left(1+\frac{h_3}{3}+\frac{h_5}{5}+\ldots\right)$$

$$\frac{B}{B_0}=\frac{\mathsf{E}}{\mathsf{E}_0}=\frac{A}{A_0}\left(1+\frac{h_3}{3}+\frac{h_5}{5}+\ldots\right)=\frac{1+\frac{h_3}{3}+\frac{h_5}{5}+\ldots}{\sqrt{1+h_3^2+h_5^2+h_7^2+\ldots}}$$

The form factor has the value

$$f=\frac{E}{\mathsf{E}}=\frac{\pi}{2\sqrt{2}}\frac{\sqrt{1+h_3^2+h_5^2+\ldots}}{\left(1+\frac{h_3}{3}+\frac{h_5}{5}+\ldots\right)}$$

and for a sine wave

$$f_0=\frac{E_0}{\mathsf{E}_0}=\frac{\pi}{2\sqrt{2}}$$

If any of the harmonics are reversed in phase, $\theta = 180°$, and the corresponding term in the expression for E is negative. This case can be included in the case where $\theta = 0$ by considering negative values for h.

One Harmonic.

Let us consider the case where a single harmonic is present.

$$E = \frac{A}{\sqrt{2}}\sqrt{1+h^2}$$

$$\mathsf{E} = \frac{2A}{\pi}\left(\cos\ \phi_1 + \frac{h}{m}\ \cos\ (m\phi_1 + \theta)\right)$$

If the harmonic be in phase with the fundamental

$$\mathsf{E} = \frac{2A}{\pi}\left(1 + \frac{h}{m}\right)$$

$$\frac{B}{B_0} = \frac{\mathsf{E}}{\mathsf{E}_0} = \frac{1+\frac{h}{m}}{\sqrt{1+h^2}}$$

If h be negative, or the harmonic reversed, the loss will always be less than with a sine wave. This corresponds to a peaked wave for $m=3$, $m=7$, $m=11$, etc.

If h be positive, the loss may be greater or less according as $1+\frac{h}{m}$ is greater or less than $\sqrt{1+h^2}$, and the loss will be unchanged when these two expressions are equal. For small values of h, $1+\frac{h}{m}$ is greater, and the loss is increased. With increasing values of h the loss reaches a maximum, decreases again to the value for a sine wave, and then goes lower.

The value of h which leaves the loss unchanged is found by equating the numerator and denominator of the expression for $\frac{B}{B_0}$

$$1+\frac{h}{m} = \sqrt{1+h^2}$$

from which

$$h = \frac{2m}{m^2-1}$$

For maximum loss the derivative with respect to h will be zero.

$$\frac{\partial\left(\dfrac{1+\dfrac{h}{m}}{\sqrt{1+h^2}}\right)}{\partial h}=\frac{1}{m}\frac{1}{\sqrt{1+h^2}}-\left(1+\frac{h}{m}\right)\frac{h}{(1+h^2)^{3/2}}=0$$

which gives

$$h=\frac{1}{m}$$

For large values of m, the maximum occurs for a value of h which is one-half the value for no change, and in any case at approximately one-half.

The values of h which give a maximum loss and which give the same loss as a sine wave, are to be found in the adjoining Table I.

TABLE I.

m	h		Maximum Value of $\left(\frac{E}{E_0}\right)^{1.6}$
	For Maximum Loss	For Unchanged Loss	
3	0.333	0.750	1.088
5	.200	.417	1.032
7	.143	.292	1.016
9	.111	.225	1.010
11	.091	.183	1.007
13	.077	.155	1.005
15	.067	.134	1.0035

The maximum value of the loss depends upon $\left(\frac{E}{E_0}\right)^{1.6}$, which may be obtained by substituting the value of h for the maximum and expanding.

$$\left(\frac{E}{E_0}\right)^{1.6}=\left(\frac{1+\dfrac{h}{m}}{\sqrt{1+h^2}}\right)^{1.6}=\left(\frac{1+\dfrac{1}{m^2}}{\sqrt{1+\dfrac{1}{m^2}}}\right)^{1.6}=\left(\sqrt{1+\frac{1}{m^2}}\right)^{1.6}=\left(1+\frac{1}{m^2}\right)^{0.8}$$

$$=1+\frac{0.8}{m^2}-\frac{0.08}{m^4}+\frac{0.032}{m^6}-\cdots\cdots\cdots$$

These values also are given in Table I.

In Fig. 1 the values of $\left(\frac{E}{E_0}\right)^{1.6}-1$ are plotted for various values of h and for the harmonics from $m=3$ to $m=15$. Two sets of curves are drawn, one for $\theta=0$ and one for $\theta=180°$. An ordinate of one of these curves multiplied by q is the change in iron loss for the given case in terms of the loss for sine wave.

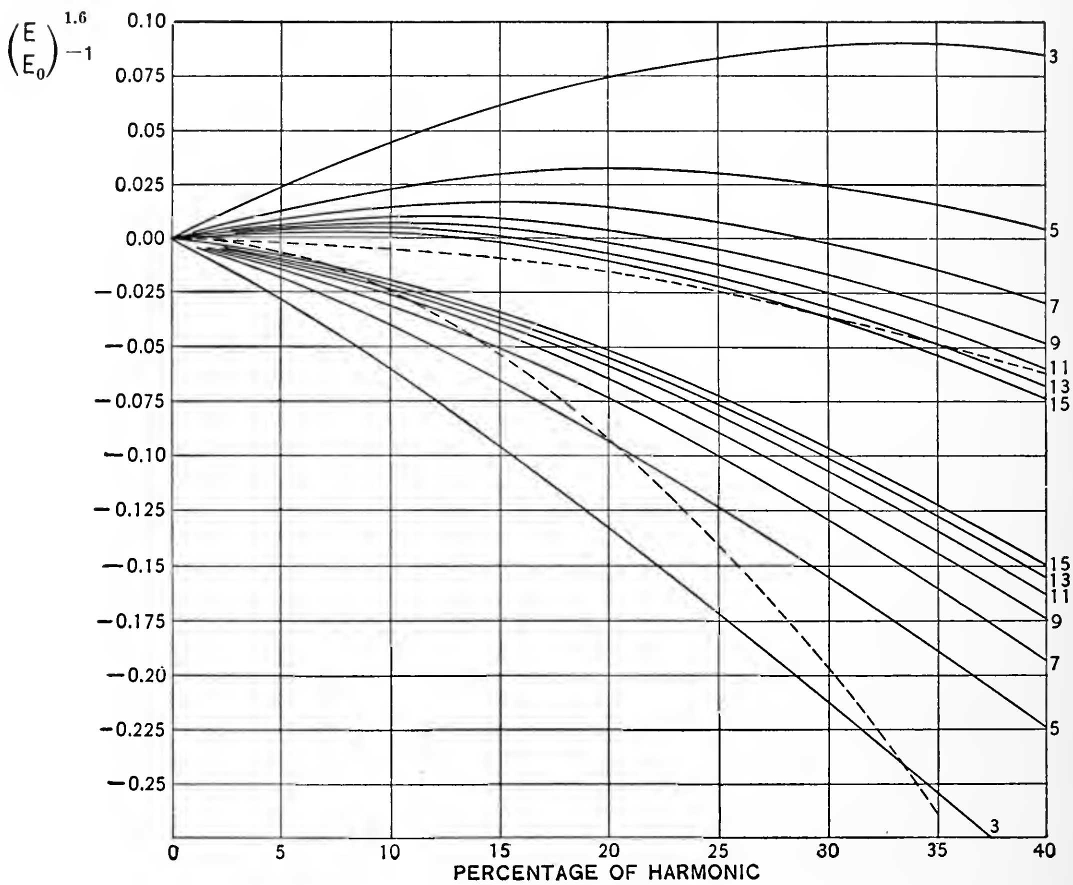

Fig. 1.

If we give h the same value in the case of different harmonics, the loss depends upon m. For $\theta=0$, $\frac{B}{B_0}$ is larger, as m is smaller. But for $\theta=180°$, $\frac{B}{B_0}$ is less, as m is smaller. The two values of $\frac{B}{B_0}$ lie nearer together the higher the order of the harmonic. The curves of Fig. 1 illustrate this very clearly.

If h be proportional to m or $h=km$

$$\frac{B}{B_0}=\frac{1+k}{\sqrt{1+m^2k^2}}$$

and the loss is less the greater is m. For small values of k the decrease is nearly proportional to m^2.

If h do not exceed the value for no change in iron loss when $\theta=0$, there will be some value of θ which will leave the loss unchanged; for in changing θ from 0° to 180° the difference in loss changes from an increase to a decrease.

$$\frac{E}{E_0}=\frac{1}{\sqrt{1+h^2}}\left[\cos\phi_1+\frac{h}{m}\cos(m\,\phi_1+\theta)\right]$$

In this case $E=E_0$ and hence

$$\sqrt{1+h^2}=\cos\phi_1+\frac{h}{m}\cos(m\,\phi_1+\theta)$$

By definition $e=0$ when $\phi=\phi_1$ or

$$\sin\phi_1+h\sin(m\,\phi_1+\theta)=0$$

These two equations are sufficient to determine ϕ_1 and θ. The form of the equations makes it easier to solve for $\cos\phi_1$ or $\cos(m\,\phi_1+\theta)$ rather than for $\sin\phi_1$. Since ϕ_1 is a small angle, greater accuracy is attained by solving for $\cos(m\,\phi_1+\theta)$.

Let

$$\sqrt{1+h^2}=\mu$$

$$m\,\phi_1+\theta=v$$

Then

$$\mu=\cos\phi_1+\frac{h}{m}\cos v$$

$$\cos^2\phi_1=\left(\mu-\frac{h}{m}\cos v\right)^2$$

Also

$$\sin\phi_1=-h\sin v$$

$$1-\cos^2\phi_1=h^2\sin^2 v=h^2(1-\cos^2 v)$$

Eliminating $\cos\phi_1$ and solving for $\cos v$ we get

$$\cos v=\frac{m}{h(m^2-1)}\left[-\mu+\sqrt{h^2(2m^2-1)+1}\right]$$

Substituting numerical values will give v; ϕ_1 is then obtained from

$$\sin\phi_1=-h\sin v$$

and finally θ from $m\,\phi_1+\theta=v$.

If the proportion of harmonic varies inversely as its order, v is roughly constant.

For let $mh=k$

Then

$$\cos v = \frac{m^2}{k(m^2-1)}\left[-\frac{\sqrt{m^2+k^2}}{m}+\sqrt{\frac{k^2}{m^2}(2m^2-1)+1}\right]$$

$$=\frac{1}{k\left(1-\frac{1}{m^2}\right)}\left[-\sqrt{1+\frac{k^2}{m^2}}+\sqrt{2k^2+1-\frac{k^2}{m^2}}\right]$$

As m increases, the value of the bracket increases and the value of the coefficient decreases, both in small degree; the largest change is in the coefficient, and may amount to about 10 per cent.

We see on inspection also that if h be small, θ is about 90°. For $\theta=v-m\,\phi_1$ and ϕ_1 is of opposite sign to v, cos v in this case being a small quantity and v consequently a large angle. The larger is m, the smaller must h be to give the same value of θ.

To illustrate by a numerical example let $m=3$, $h=0.20$

Then

$$\cos v=\frac{3}{1.6}(-1.0198\pm1.2962)=0.51825 \text{ or } -4.34$$

$$v=\pm58°47'.5 \qquad \phi_1=\mp9°51'$$

$$\theta=\pm88°20'$$

Again, let $m=5$, $h=0.12$

$$\cos v=\frac{5}{2.88}[-1.0072\pm1.3057]=0.5188$$

$$v=\pm58°45' \qquad \phi_1=\mp5°53'.3$$

$$\theta=\pm88°11'.5$$

For the case of $m=3$ and various values of h, the values of θ have been worked out as shown in Table II and plotted in Fig. 2. This curve shows how θ decreases from 90° to 0° as h increases from zero to the limiting value given in Table I. If h exceed this value, the loss will be decreased for every value of θ. The curve resembles closely an ellipse, but is somewhat fuller than either an ellipse or a cycloid with the same semiaxes. The curve has no simple equation, as may be seen by reference to the expressions used in calculating its coordinates.

Other curves have been plotted in Fig. 2, each connecting points representing the same loss, the value of the loss being different for

each locus. Each line represents a change of two per cent in the value of $\left(\frac{E}{E_0}\right)^{1.6}$; the lines inside the "no-change" curve representing increased loss, those outside decreased loss.

TABLE II.

Calculation of phase angle for unchanged loss with third harmonic.

h	μ	cos v	v	sin v	sin ϕ_1	ϕ_1	θ
0.0	1.0000	0.0000	90° 0′	1.0000	0.0000	0° 0′	90° 0′
0.1	1.0050	0.2876	73° 17′	0.9577	0.0958	5° 30′	89° 47′
0.2	1.0198	0.5182	58° 48′	0.8553	0.1711	9° 51′	88° 20′
0.3	1.0440	0.6832	46° 54′	0.7302	0.2191	12° 39′	84° 51′
0.4	1.0770	0.7985	37° 1′	0.6020	0.2408	13° 56′	78° 49′
0.5	1.1180	0.8800	28° 22′	0.4750	0.2375	13° 44′	69° 34′
0.55	1.1413	0.9117	24° 16′	0.4108	0.2260	13° 4′	63° 27′
0.6	1.1662	0.9388	20° 9′	0.3445	0.2067	11° 56′	55° 57′
0.65	1.1927	0.9622	15° 48′	0.2723	0.1770	10° 12′	46° 24′
0.7	1.2207	0.9824	10° 46′	0.1869	0.1308	7° 31′	33° 19′
0.75	1.2500	1.0000	0° 0′	0.0000	0.0000	0° 0′	0° 0′

So long as the emf. passes through the zero value only twice per cycle, there is little difficulty in determining the value of ϕ_1. It is only necessary to find where $e=0$. If, however, the emf. cross the axis more than this number of times, it becomes necessary to discriminate between the different values of ϕ for which $e=0$. This takes place only in cases where there are secondary maxima in the flux curve, and it is of course the highest maximum which must be located. For any half wave of emf. the number of crossings will be an odd number, and they are alternately in the same direction as the fundamental wave and in the opposite direction. Those in the opposite direction represent minima, and this class includes the case $\phi=0$ for $\theta=180°$ with a large component of harmonic. In the previous work we have taken $\phi_1=0$ for $\theta=180°$, but this holds only when there is but one value of ϕ for which $e=0$ in that half wave. The limiting values of h to avoid secondary maxima in the wave of magnetic flux are given in Table III for $\theta=0$ and $\theta=180°$. For other values of θ, the limiting values of h may be taken from the curves of Fig. 3.

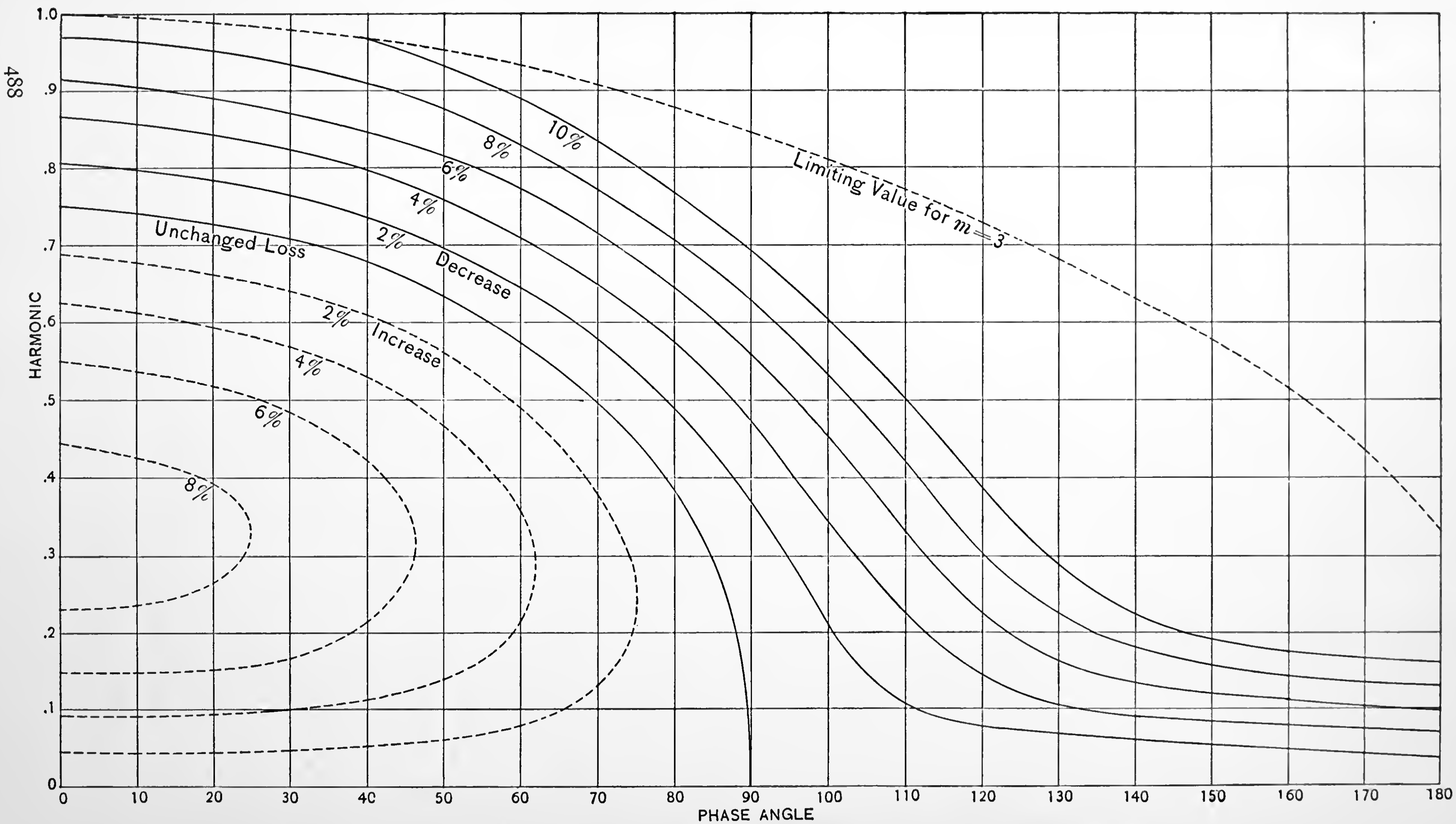

Fig. 2.—*Showing the change in hysteresis loss for the third harmonic, w th different percentages and phase angles.*

TABLE III.

m	Limiting Values of h	
	$\theta = 0°$	$\theta = 180°$
3	1.000	0.333
5	.800	.200
7	.613	.143
9	.490	.111
11	.407	.091
13	.347	.077
15	.302	.067

It will be noticed that the limiting values of h for $\theta = 180°$ are the same values which give a maximum loss when $\theta = 0$; viz, $h = \frac{1}{m}$.

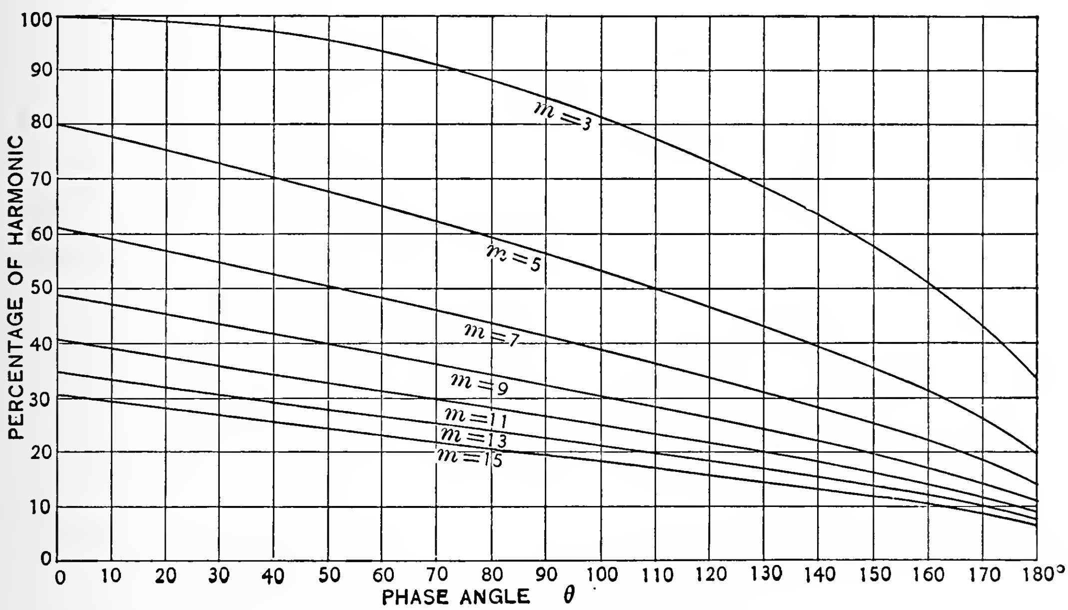

Fig. 3.—*Limiting Values of Harmonic.*

The value of ϕ_1 is to be chosen from those values of ϕ for which the emf. curve crosses the axis in the same direction as the fundamental, and that value of ϕ should be taken which will give the largest average emf. If the curve of the emf. has been plotted, it can often be seen from inspection which is the desired value. Thus in Fig. 4 the average should be taken between a and d, which gives two larger lobes positive and the smallest lobe negative, since it is obvious that this will be larger than the average between c and

f, which includes a negative lobe of medium size. The average between b and c is yet smaller and represents a minimum in the curve of magnetic flux.

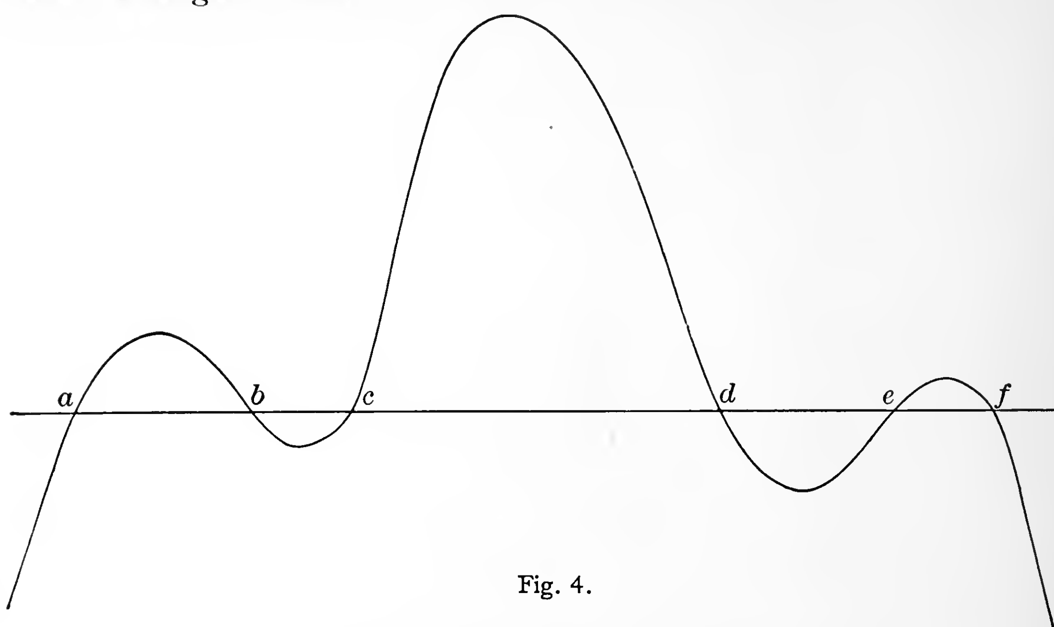

Fig. 4.

In the case of a large component of high harmonic the value of ϕ_1 can be determined approximately from inspection. The harmonic will pass through zero every time ϕ changes by $\frac{\pi}{m}$, and one of these zero values will be at $-\frac{\theta}{m}=\phi$. This will be very close to the value of ϕ for which $e=0$ and for which $\phi=\phi_1$, but will be slightly further from $\phi=0$. This results because the fundamental has relatively small values in this neighborhood and the harmonic is changing very rapidly.

We may say then

$$\mathsf{E}=\frac{2A}{\pi}\left[\cos\frac{\theta}{m}+\frac{h}{m}\cos\left(-m\frac{\theta}{m}+\theta\right)\right]$$

$$=\frac{2A}{\pi}\left[\cos\frac{\theta}{m}+\frac{h}{m}\right]$$

The extreme values of E occur for $\theta=0$ and $\theta=180°$, and this approximation includes the extreme values of the true E. ϕ_1 is always zero for $\theta=0$, and in that case

$$\mathsf{E}=\frac{2A}{\pi}\left(1+\frac{h}{m}\right)$$

as in the original derivation. The range of variation is from 1 to $\cos\frac{\theta}{m}$, and this is smaller, the larger is m.

If we let $m=15$, $\theta=180°$

$$\mathsf{E}=\frac{2A}{\pi}\left(\cos 12°+\frac{h}{m}\right)=\frac{2A}{\pi}\left(0.978+\frac{h}{m}\right)$$

or the range of variation in E is about 2.2 per cent.

If $m=13$ $\theta=180°$

$$\mathsf{E}=\frac{2A}{\pi}\left(\cos 13°51'+\frac{h}{m}\right)=\frac{2A}{\pi}\left(0.971+\frac{h}{m}\right)$$

a range of nearly three per cent.

We see, then, that with a large percentage of high harmonic the loss is always decreased, and the amount of decrease varies only slightly with the phase angle.

There is another effect which must be considered when the emf. passes through zero more than once per half cycle, that is, when the wave of magnetic flux has more than one maximum. We have assumed that the hysteresis is proportional to the 1.6th power of the maximum flux density. The assumption is at least approximately true when the iron is put through a simple cycle, but no validity is claimed for it in cases where the magnetic flux does not progress continuously from a positive maximum to a negative maximum. If there be more than one maximum to the half cycle, there will be an extra or secondary loop in the hysteresis curve, and greater power will be consumed. The formula worked out does not strictly apply to such cases. A quantitative examination of actual cases shows, however, that this extra power will seldom amount to as much as one per cent of the whole, and consequently may usually be neglected. Only when one of the harmonics enters to the same order of magnitude as the fundamental does this source of error become important. Thus with fifty per cent of the fifth harmonic the extra loop is through a range of only four per cent of B, and this would not add more than one per cent to the hysteresis loss.

Figure 1 is based upon values of E taken for $\phi_1=0$, and consequently the curves do not represent $\left(\frac{B}{B_0}\right)^{1.6}$, except within the limits

given by Table III. These limits are shown on the figure by the dotted lines. For values of h only slightly in excess of the limiting values, however, the effect upon E and the extra loops of hysteresis are both negligible, and the curves may be used somewhat outside of these limits.

For higher values of h the actual loss would be greater than that calculated by use of the curves, and the discrepancies will be greatest for $\theta = 180°$.

Numerical Example with one Harmonic.

When q has been determined for a given frequency, the change in iron loss may be determined for $\theta = 0°$ or $\theta = 180°$ by reference to the curves of Fig. 1. For other values of θ the value of ϕ_1 must be determined before B can be calculated. This can be done by a graphical method, or by successive trial and calculation.

Thus let $m = 3$, $h = 0.20$, $q = 0.60$

For $\theta = 0$, or a flat voltage wave,

$$\left(\frac{1+\frac{h}{m}}{\sqrt{1+h^2}}\right)^{1.6} = 1.075$$

$$\frac{W - W_0}{W_0} = 0.045$$

an increase of 4.5 per cent.

For $\theta = 180°$, or peaked voltage wave

$$\left(\frac{1-\frac{h}{m}}{\sqrt{1+h^2}}\right)^{1.6} = 0.867$$

$$\frac{W - W_0}{W_0} = -0.080$$

a decrease of 8.0 per cent.

The results for intermediate values of θ are given in Table IV.

TABLE IV.

θ	ϕ_1	$S=\frac{\pi E}{2A}$	$\left(\frac{S}{\sqrt{1.04}}\right)^{1.6}$	$\frac{W-W_0}{W_0}$ in Per Cent
0°	0°	1.067	1.075	+ 4.5
± 30	∓ 3.7	1.061	1.065	+ 3.9
± 60	∓ 7.15	1.044	1.038	+ 2.3
± 90	∓ 10.0	1.018	0.997	— 0.2
± 120	∓ 11.5	0.985	0.946	— 3.2
± 150	∓ 10.0	0.951	0.895	— 6.3
± 180	0.	0.933	0.867	— 8.0

Two Harmonics.

In this case

$$E=\frac{2A}{\pi}\left[\cos\phi_1+\frac{h_1}{m_1}\cos(m_1\phi_1+\theta_1)+\frac{h_2}{m_2}\cos(m_2\phi_1+\theta_2)\right]$$

$$\frac{A}{A_0}=\frac{1}{\sqrt{1+h_1^2+h_2^2}}$$

If the harmonics be not in phase with the fundamental, any special numerical case can be worked out by finding the value of ϕ_1 corresponding to definite values of θ_1 and θ_2.

If the harmonics be assumed in phase with the fundamental, $\phi_1=0$ and

$$E=\frac{2A}{\pi}\left(1+\frac{h_1}{m_1}+\frac{h_2}{m_2}\right)$$

$$\frac{B}{B_0}=\frac{1+\frac{h_1}{m_1}+\frac{h_2}{m_2}}{\sqrt{1+h_1^2+h_2^2}}$$

If we let $h_1=h_2$

$$\frac{B}{B_0}=\frac{1+h\left(\frac{1}{m_1}+\frac{1}{m_2}\right)}{\sqrt{1+2h^2}}$$

and there will be some value of h which will leave the loss unchanged. Let $m_1=3$, $m_2=5$, then

$$1+h\left(\frac{1}{3}+\frac{1}{5}\right)=\sqrt{1+2h^2}$$

from which $h=0.622$.

Again, if the fifth harmonic be reversed in phase

$$1+h\left(\frac{1}{3}-\frac{1}{5}\right)=\sqrt{1+2h^2}$$

and h = 0.1346.

Again, we may assign a definite value to h_1 and find the value of h_2 which leaves the loss unchanged. Let $h_1=0.2$, $m_1=3$, $m_2=5$.

$$1+\frac{0.2}{3}+\frac{h_2}{5}=\sqrt{1+0.04+h_2^2}$$

from which $h=0.611$ or -0.167, indicating that the fifth harmonic may be introduced in reversed phase with magnitude one-sixth of the fundamental, or in same phase with magnitude 61 per cent of the fundamental.

Several Harmonics.

Going back to the general expression for E, we have

$$\frac{B}{B_0}=\frac{\left[\cos\phi_1+\frac{h_3}{3}\cos(3\phi_1+\theta_3)+\frac{h_5}{5}\cos(5\phi_1+\theta_5)+\ldots\ldots\right]}{\sqrt{1+h_3^2+h_5^2+h_7^2+\ldots\ldots}}$$

The numerator of this expression may be either greater or less than unity, depending upon the values of θ_3, θ_5, etc. The denominator is independent of the phase relations and is never less than unity. We may consider the influence of the two factors separately. The denominator always tends to decrease the loss; for small values of h by an amount proportional to h^2 and for larger values by an ever decreasing proportion. Superposed upon this decrease is the effect of the numerator. When the numerator is greater than unity, it may overpower the denominator for small values of h, but can not do so for the larger values. If a number of harmonics are introduced at random, the probability is that the loss will be decreased, and the greater the number of harmonics, the greater the probable decrease. For with various phase angles some of the terms in the numerator will counteract others, while in the denominator all work together for a decrease.

In special cases the higher harmonics may be present in greater magnitude than the lower ones, but more often their amplitude is

small, and we will consider the case where h is inversely proportional to the order of the harmonic.

Let $k=3h_3=5h_5=7h_7$, etc., $\phi_1=0$, $\theta=0$ or $180°$.
Then

$$\frac{B}{B_0}=\frac{1+k\left(\pm\frac{1}{3^2}\pm\frac{1}{5^2}\pm\frac{1}{7^2}\pm\ \dots\right)}{\sqrt{1+k^2\left(\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ \dots\right)}}$$

If $k<1$, an accuracy of one per cent will be attained by disregarding harmonics above the ninth. If each θ be zero,

$$\frac{B}{B_0}=\frac{1+k\left(\frac{1}{9}+\frac{1}{25}+\frac{1}{49}+\ \dots\right)}{\sqrt{1+k^2\left(\frac{1}{9}+\frac{1}{25}+\frac{1}{49}+\ \dots\right)}}$$

If in this case we let $k=0.5$, $q=0.6$

$$\frac{B}{B_0}=1.074 \qquad \frac{W-W_0}{W_0}=0.071$$

It is readily seen that the lower harmonics and especially the third will be predominant, unless it be introduced with θ near $90°$, and even then its influence in the denominator is unimpaired.

It is to be noticed that the expression for $\frac{B}{B_0}$ also represents the ratio $\frac{f_0}{f}$; in other words, an increase or decrease in loss depends upon whether the form factor of the wave of voltage is less or greater than the form factor of a sine wave. In the special cases previously considered, where the loss is unchanged, the form factor has the same value as for a sine wave, namely, $\frac{\pi}{2\sqrt{2}}=1.1107$.

We may express the percentage change in loss as

$\frac{W-W_0}{W_0}=q\left[\left(\frac{f_0}{f}\right)^{1.6}-1\right]$ or $1.6\, q\, \frac{f_0-f}{f}$ when f_0-f is small compared to f. It is not necessary, therefore, to know the components of the wave used in order to calculate the loss. *It is sufficient to determine the form factor.* This can always be done if apparatus is

available for tracing the wave[3], and the more laborious process of analyzing the wave may be omitted.

If we define a "peaked" wave to be one whose form factor is greater than 1.1107 and a "flat" wave to be one whose form factor is less than 1.1107, then we may make the general statement that a peaked emf. wave reduces the iron loss of a transformer and a flat wave increases the iron loss. If we have a third harmonic combined with the fundamental, the peaked wave occurs when $\theta = 180°$ or with the maxima of the two components coinciding in time. With the fifth harmonic we again have a peaked wave for $\theta = 180°$, but in this case the positive maximum of the fundamental is coincident with the negative maximum of the harmonic, and from mere inspection

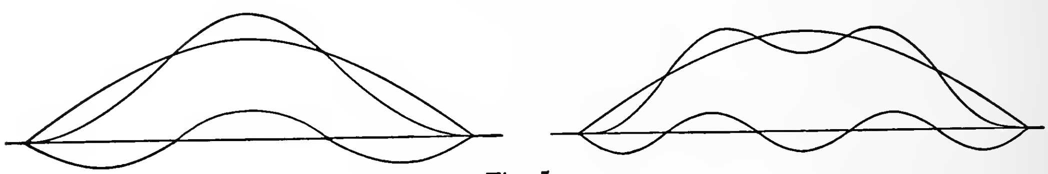

Fig. 5.

the curve would hardly be classed as peaked. These two cases are illustrated in Fig. 5. It is not desirable, consequently, to define a "peaked" wave as one having coincident maxima for fundamental and harmonic, for then "peaked" and "flat" would cease to be discriminating terms. Defining the wave by means of the form factor, its nature is determined largely by the value of θ, and if a single harmonic be present, the wave will always be peaked when $90° < \theta < 180°$ and will usually be flat when θ is near zero. This can usually be judged upon inspection by the steepness of the curve where it passes through zero.

2. DESCRIPTION OF APPARATUS.

The power expended in iron losses was measured by means of a dynamometer wattmeter.[4] The fixed coils are in series connection with the primary winding of the transformer; the movable coils are connected, with a suitable multiplier, to the terminals of the

[3] The form factor can also be determined directly if apparatus is available for measuring the average value of the emf. Such an apparatus is described by Lloyd and Fisher, this Bulletin, p. 501, and by Rose and Kühns, E. T. Z., **24**, p. 992; 1903.

[4] This instrument is of the type described in this Bulletin, **1**, p. 424, by Rosa, Lloyd, and Reid.

secondary winding. If the secondary contain the same number of turns as the primary, and enclose the same magnetic flux, the wattmeter then measures the energy supplied to the core and to the secondary circuit or circuits. If the turns be not the same in number, their ratio gives the factor connecting the wattmeter reading with the true watts. As only relative values are desired in the present investigation, it is unnecessary to know this factor. This method has an advantage over the more usual method of applying the primary voltage to the potential circuit of the wattmeter, since the copper loss in the primary is not included in the measurement. Usually this must be determined and applied as a correction.

To get the iron losses alone, a correction must be applied for the energy supplied to the secondary circuits. The energy expended in a noninductive secondary is equal to the square of the effective voltage induced in that secondary divided by the resistance of the secondary circuit. In the present work this correction is small and constant, and may be neglected in considering relative losses.

The induced voltage was measured by a dynamometer voltmeter which in most cases was connected to a secondary winding. This instrument measures the effective voltage and its deflection was kept constant during a series of runs. Its inductance was known, and the effect of high frequency currents was compensated by slight changes in its noninductive multiplier. In some of the runs this voltmeter was connected across the primary terminals, notably when the secondary voltage was low and the component of high harmonic large, as in these cases the voltmeter correction would otherwise become very large. This method of connection keeps the *applied* effective voltage constant and thus more nearly fulfils the conditions of commercial operation. It does not keep the *induced* voltage so constant, however, and thus does not so nearly realize the conditions assumed in the theoretical deductions. In the cases where both methods of connection were used in turn, the difference in the results was less than one per cent.

A Thomson ammeter of the magnetic vane type was inserted in the primary circuit and readings of the magnetizing current were made. No use of these readings is made in the present work. The resistance of this ammeter is 0.4 ohm, of the field circuit of wattmeter 1.7 ohm, a total of 2.1 ohms in series with the primary.

No rheostat was used in this circuit, the applied voltage being controlled through the field excitation of the generators. As the primary current in most cases was about one ampere, the ohmic drop in the instruments was only about two volts.

The connections are shown in the diagram, Fig. 6.

When the transformer used had two secondaries, the voltmeter and potential circuit of wattmeter were connected to separate windings. When there was a single secondary, the two were connected in parallel to its terminals. The deflections of the wattmeter were taken as proportional to the watts, when the multiplier R remains constant. This is true for deflections not differing more than a few

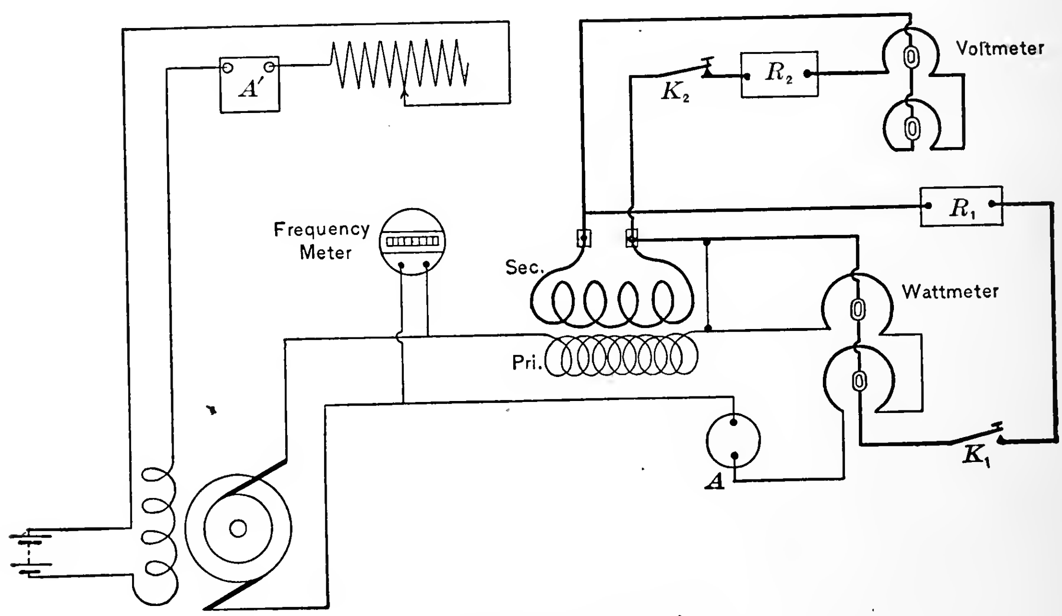

Fig. 6.—*Diagram of Connections.*

centimeters, as is the case in these measurements. In determining q, the multiplier R was changed to give the same wattmeter deflection on both frequencies, and the watts taken to be proportional to the resistance of the potential circuit. Calibration of the instrument was therefore unnecessary.

The frequency of the generator was measured by a Hartmann-Kempf frequency meter of the new type, containing an indicator for each half cycle. By reference to this instrument the frequency could be read with greater accuracy than ¼ cycle, which was considered sufficient. Sixty cycles was the frequency used in the measurements, and the proper speed of the generator was maintained by adjustment of the field resistance of the driving motor.

As this motor was supplied with current from a storage battery, the changes in speed were gradual, and the necessary adjustments slight and infrequent.

The transformers used were:

1. General Electric Co., Type H, No. 290645, 120/4 volts, 60 cycles, 2,000 watts, $q=0.61$.

2. Siemens and Halske No. 6700, 120/2 volts, 50 cycles, 1,600 watts, $q=0.60$.

3. General Electric Co., Type H, No. 290414, 480–240–120/30–60–120 volts, 60 cycles, 500 watts, $q=0.70$.

Transformer No. 1 was operated at 110 volts, and No. 2 at 132 volts, 60 cycles being used throughout. Otherwise the conditions were normal, and the transformers were operated under no load. In this case the ratio of transformation is not influenced by the wave-form.[5]

No. 3. has four separate primary windings and four separate secondaries, which can be connected in series or parallel, thus permitting various ratios of transformation.

3. OBSERVATIONS.

When starting a series of observations, the primary voltage was adjusted to the desired value on sine wave by reference to a calibrated portable voltmeter. The reading of the volt-dynamometer was taken at the same time, and the same deflection was maintained throughout the series by adjustment of rheostat in generator field. Readings were then taken on the wattmeter with sine wave, and followed by readings with other wave forms. At frequent intervals throughout the series the sine wave was again used, and control thus kept of the changes due to heating. If current be kept on, the iron loss continually decreases on account of the heating, and a reading on any wave form must be compared with the mean of sine wave readings before and after. Every result given is the mean of three observations taken in immediate succession, the voltmeter being set independently for each. At the beginning or end of the series observations were taken at 30 cycles, in order to determine the proportion of hysteresis to the total iron loss. For this purpose the multiplier of the voltmeter was changed so as to make the total

[5] G. Roessler, Electrician, **36**, p. 151; 1895.

resistance of its circuit just one-half of the previous value, and adjustment made for the same deflection. The voltage must then also be one-half of the previous value.

In using distorted waves which involve higher frequencies the voltmeter must be corrected for its inductance error. The correction is easily made for a sine wave, and most conveniently by altering the resistance so that the impedance remains constant. Let L be the inductance of the instrument. Then, if the multiplier be noninductive and R be the total resistance, the impedance is

$$\sqrt{R^2+4\pi^2n^2L^2}=R\left(1+\frac{4\pi^2n^2L^2}{R^2}\right)^{\frac{1}{2}}$$

To have the impedance the same as with direct current, the resistance must be changed to $R'=R-\frac{2\pi^2n^2L^2}{R}$ (L is very small compared to R, its value being 0.138 henry.) With a distorted wave the resistance required by the different components is different, but by giving proper weight to each component (i. e., in proportion to the square of the amplitude) we get

$$R-R'=\frac{2\pi^2n^2L^2}{R}(1+h_1^2m_1^2+h_2^2m_2^2+\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$

Hence, in changing from the sine wave to a distorted wave, we change the resistance of the multiplier by

$$\frac{2\pi^2n^2L^2}{R}(h_1^2m_1^2+h_2^2m^2{}_2+\ .\ .\ .\ .\ )$$

To determine the components of the wave used, the magnetizing circuit is broken after adjustments have been made, and the voltage of each generator is measured with a Stanley hot wire voltmeter. This is done on no load to avoid the drop on the line. As the load is very small compared to the capacity of the generators, the armature reaction does not appreciably affect the voltage.

The procedure in taking observations is illustrated by a run taken to determine the dependence of the result upon the way of connecting the voltmeter. The readings are given in Table V.

Each of the deflections given for voltmeter and wattmeter is the mean of three observations. The calculated effects are submitted

TABLE V.

Effect of Voltmeter Connection.

Transformer No. 2. q=0.60, m=3

h	θ	Voltmeter			Wattmeter		
		Resistance	Deflection	Connection	Deflection	Increase due to Harmonic	Increase in Per Cent
0.		25000	27.95	Primary (132 Volts)			
0.		25000	27.95	Primary	20.16		
0.23	180°	25000	27.95	"	18.25	−1.80	−9.0
0.23	0°	25000	27.95	"	21.01	+0.88	+4.4
0.		25000	27.95	"	20.12		
0.		500	19.35	Secondary (132 Volts on Primary)			
0.		500	19.35	Secondary	19.77		
0.23	180°	499	19.35	"	17.84	−1.91	−9.7
0.23	0°	499	19.35	"	20.61	+0.89	+4.5
0.		500	19.35	"	19.70		
0.23	180°			Calculated			−9.3
0.23	0°						+4.8

for comparison. They are readily obtained from Fig. 1. A similar set with transformer No. 1, using 21 per cent of fifth harmonic, gave −0.8 per cent and −3.0 per cent, with voltmeter on primary; −0.9 per cent and −3.4 per cent, with voltmeter on secondary.

The agreement with each other and with the calculated values are seen to be well within one per cent, and this is about the order of discrepancies throughout the work.

The value of q for the above transformer was determined from the observations given in Table VI, with the voltmeter connected to secondary.

The observations were made with the help of Mr. J. V. S. Fisher, to whom credit is also due for assistance in the computations and drawings.

4. EXPERIMENTAL RESULTS.

Table VII gives the results on two transformers of introducing a single harmonic either in phase or reversed. For comparison the calculated effects taken from Fig. 1 are added to the Table, those marked with an asterisk lying outside the limits set by the theoretical conditions.

TABLE VI.

Transformer No. 2. Separation of Hysteresis and Eddy Current Losses

n	Voltmeter		Wattmeter		$\frac{R_n}{n}$	Hysteresis	Eddy Currents	q
	Res.	Defl.	Res.	Defl.				
60	500	19.31	900	20.05	15.00	8.94	6.06	.596
30	250	19.31	359	20.04	11.97	8.94	3.03	.747

Table VIII gives the results of shifting the phase of the harmonic, while keeping its magnitude constant, and these are plotted in the curves of Fig. 7. The calculated values in Table IV may be compared with the corresponding column of Table VIII, viz, for transformer No. 2, $m=3$.

TABLE VII.

Transformer	m	h	Percentage Increase in Loss			
			$\theta=0^\circ$		$\theta=180^\circ$	
			Exp.	Calc.	Exp.	Calc.
No. 1	3	.11	+2.8	+3.0	−4.0	−4.1
" "	5	.105	+1.7	+1.5	−2.8	−2.6
" "	7	.19	+0.7	+0.9	−3.8	−4.3 *
" "	7	.235	−0.2	+0.6	−5.2	−5.6 *
" "	7	.26	−0.4	+0.4	−5.7	−6.6 *
" "	9	.20	−0.9	+0.2	−3.3	−4.0 *
" "	11	.12	+0.1	+0.4	−1.0	−1.8 *
" "	13	.125	−0.4	+0.1	−1.1	−1.7 *
" "	15	.14	−0.9	−0.1	−1.5	−1.9 *
No. 2	3	.20	+4.0	+4.5	−8.3	−7.9
" "	5	.115	+1.2	+1.6	−3.3	−2.9
" "	7	.17	+0.5	+0.9	−3.7	−3.5 *
" "	7	.24	−0.3	+0.6	−5.2	−5.7 *
" "	9	.17	−0.3	+0.4	−3.1	−3.1 *
" "	11	.12	−0.3	+0.4	−2.0	−1.7 *
" "	13	.12	−0.4	+0.1	−1.1	−1.6 *
" "	15	.135	−0.5	0.0	−1.4	−1.7 *

The generators producing the high frequencies do not give very high voltages. In order to use a high percentage of these, the generator supplying the fundamental frequency was excited to a correspondingly low voltage, and after combination the voltage was transformed up to the proper value. Transformer No. 2 was

TABLE VIII.

Per Cent Increase in Loss

Transformer	No. 1	No. 2	No. 2	No. 2	No. 3	No. 3	No. 3
m	3	3	5	7	3	5	15
h	.18	.20	.115	.23	.165	.115	.41
$\theta =$ 0°	+5.5	+4.0	+1.2	−0.3	+4.4	+1.5	−7.3
30	+5.7	+4.2		−0.1	+4.7		−7.0
45			+1.0			+1.5	
60	+4.8	+3.2		−0.6	+4.0		−7.0
90	+2.0	+0.8	−0.2	−1.4	+2.0	+0.6	−6.7
120	−1.6	−1.9		−2.3	−1.0		−7.0
135			−1.9			−1.0	
150	−5.1	−5.6		−3.7	−4.2		−7.2
180	−8.2	−8.3	−3.3	−5.2	−7.4	−2.7	−7.4
210	−8.1	−8.0		−5.1	−7.6		−7.5
225			−2.1			−3.0	
240	−5.6	−5.4		−3.6	−5.8		−7.8
270	−2.3	−2.3	−0.5	−2.5	−2.7	−1.2	−7.8
300	+1.2	+0.7		−1.4	+0.3		−7.8
315			+0.7			+0.5	
330	+3.9	+2.9		−0.8	+2.8		−7.6

used in this way with the results shown in Table IX. The voltmeter was connected to the primary side of the transformer. The calculated values for $\theta = 180°$ are obtained by the method given on p. 490.

It will be noticed here that the experimental and the calculated values do not agree very well; indeed, the experimental results show very little effect from phase reversal. While seeking the reason for this, the effect of transforming the fundamental alone was tried.

The iron losses were first determined by direct connection to the generator, then after transformation. With ratio of transformation 1:2 the losses were 0.7 per cent less; with ratio 1:3 they were one per cent less than by direct connection.

TABLE IX.

Transformer No. 2

Ratio of Previous Transformation.	m	h	Increase in Loss			
			$\theta=0°$		$\theta=180°$	
			Exp.	Calc.	Exp.	Calc.
1 : 2	13	.19	—1.9	—0.3	—2.3	—3.1*
1 : 3	13	.27	—4.0	—1.5	—3.5	—4.2*
1 : 2	15	.235	—2.9	—1.1	—3.1	—3.2*
1 : 3	15	.32	—5.2	—2.9*	—4.9	—4.9*
1 : 4	15	.44	—8.4	—6.0*	—8.3	—8.1*

Measurements were next made on Transformer No. 3, using the low-voltage winding as primary. In this way a large component of high harmonic could be used at a low voltage. Table X gives the results.

TABLE X.

Transformer No. 3

Ratio of Transformation	m	h	Increase in Loss in Per Cent				
			$\theta=0°$		$\theta=180°$		$\theta=90°$
			Exp.	Calc.	Exp.	Calc.	Calc.
60 : 120	13	.17	— 1.4	—0.2	— 2.3	— 3.0*	—1.6
30 : 120	13	.36	— 5.8	—3.7*	— 5.9	— 7.2*	—5.5*
30 : 120	13	.48	—10.4	—7.9*	—10.8	—11.0*	—9.5*
60 : 120	15	.155	— 1.2	—0.2	— 1.7	— 2.4*	—1.3
30 : 120	15	.32	— 4.5	—3.3*	— 4.4	— 5.7*	—4.5*

Table XI shows the effects of combining two harmonics with the fundamental. These harmonics were kept in the same phase relation with each other, and both shifted with respect to the fundamental, each shift amounting to 90° of the fundamental. The initial position was with the third harmonic in phase and the fifth harmonic reversed.

TABLE XI.

Transformer No. 2 Two Harmonics		$m_1=3$ $m_2=5$	$h_1=.113$ $h_2=.109$
Phase Setting		Increase in Loss in Per Cent	
θ_1	θ_2	Exp.	Calc.
0°	180°	−0.45	+0.3
180	0	−3.5	−2.6
− 90	− 90	−0.6	+0.8
+ 90	+ 90	+1.9	+0.8
+135	+ 45	−1.1	−0.8
− 45	−135	+0.15	+0.7

Table XII presents a case where all of the generators were used together. The phase of each harmonic in this case was made either zero or 180°, and the amplitudes decreased as the frequency increased, so that *mh* was approximately constant and had the value 0.6.

Table XIII gives the results of applying a large component of low harmonic to transformer No. 2.

5. DISCUSSION OF THE RESULTS.

Reference to Table VII discloses the fact that in the simple case of a single harmonic of small magnitude the experimental results agree with the theoretical values within one per cent. When the higher harmonics enter in large proportion the agreement is not so good; thus, with 20 per cent of the ninth harmonic the discrepancy for transformer No. 1 is 1.1 per cent. The loss with these higher harmonics in phase is always less than the calculated value; when they are reversed in phase it is nearly always greater. This is even more apparent in Tables IX and X. In the latter there is very little difference between $\theta=0°$ and $\theta=180°$, the losses having the value which is about the mean of the calculated values, and which would apply to the case $\theta=90°$.

In Table VIII and Fig. 7 it is seen that the maximum loss does not coincide with $\theta=0°$ nor the minimum with $\theta=180°$, as indicated by the theory, but that the extreme values are for $\theta=20°$ and $\theta=200°$, approximately. This can not be due to an error in the phase settings, since in the case of different harmonics different

numbers of degrees of the fundamental correspond to 20° of the harmonic. The accuracy of setting of any generator is about 0.1° of arc, corresponding to 0.3° electrical in the fundamental, 4.5° electrical in the highest harmonic, and proportional values for the intermediate harmonics. The results point rather to a distortion of the wave between the generators and the transformer; that is to say, the wave form applied to magnetization is not the same as that supplied by the generators. This may be accounted for by the ohmic drop in leads, instruments, and primary winding. It is well known that, owing to the varying permeability of the iron, the wave of current has a different shape from the wave of emf.[6] The ohmic drop of potential, consequently, involves other harmonics and other magnitudes than the generator emf. Since the emf. used to magnetize the iron is the difference of these two, it also must have a wave shape different from the generator emf. When any considerable resistance is in the circuit, this effect becomes so marked as to give an entirely different wave. It is for this reason that no rheostat was used in the primary circuit, the current being controlled instead through the excitation of the generator.

The fact that this effect is very noticeable is proven by the result obtained when an intermediate transformer was used, as compared with the losses obtained with direct connection to the generator. This amounted to one per cent with a sine curve for the original wave.

With a large percentage of high harmonic the effect is even more marked, as illustrated in the last column of Table VIII. Here the maximum occurs at $\theta = 270°$ and the minimum at $\theta = 90°$, and the range of variation is less than the theoretical value. By calculation the extreme values for this case are −5.2 per cent and −7.3 per cent as against −6.7 per cent and −7.8 per cent experimentally. In all cases of a large component of high harmonic the calculation for $\theta = 180°$ is based upon the modified formula given on p. 490, where ϕ_1 is taken as $\frac{\theta}{m}$.

With a large component of lower harmonic the results given in Table XIII show an agreement with calculated values for $\theta = 180°$ except in one case. Here the proper value for ϕ_1 has been found by

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[6] Waves illustrating this are given by H. A. Pikler, Electrical World, **42**, p. 218; 1903.

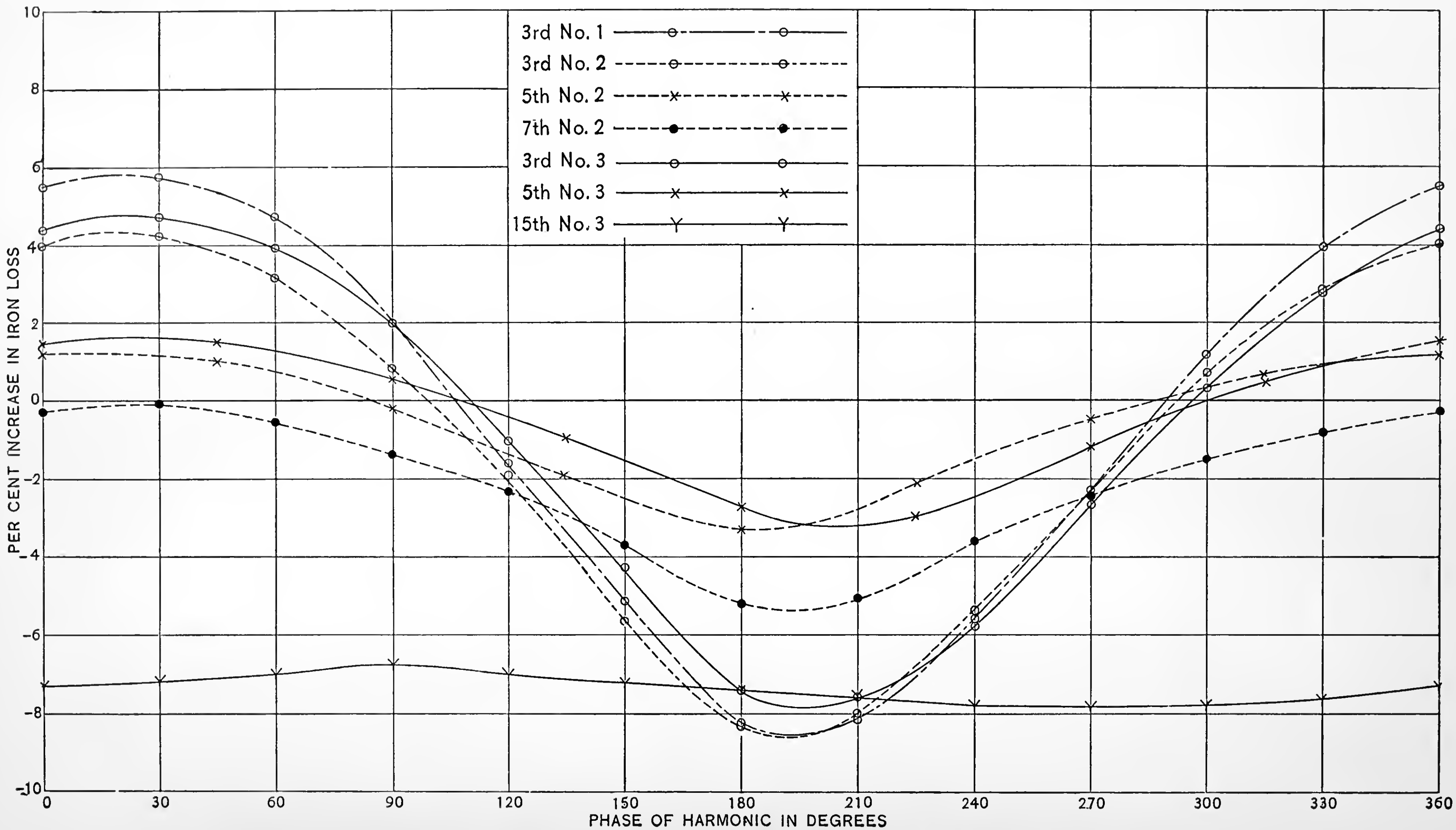

Fig. 7.—*Variation of Iron Losses with Phase of Harmonic.*

TABLE XII.

Transformer No. 2 Seven Harmonics hm=k=0.6 approximately $\theta_3=\theta_4=\theta_5=\theta_6=\theta_7$	$m_1=3$ $m_2=5$ $m_3=7$ $m_4=9$ $m_5=11$ $m_6=13$ $m_7=15$	$h_1=.20$ $h_2=.10$ $h_3=.10$ $h_4=.08$ $h_5=.065$ $h_6=.055$ $h_7=.045$

Phase Setting			Increase in Loss in Per Cent	
θ_1	θ_2	θ_3	Exp.	Calc.
180°	180°	180°	−7.8	*
0	0	0	+7.6	+7.9
0	180	180	−0.7	*
180	0	0	−5.4	−4.4
180	0	180	−7.9	*
0	180	0	+2.8	+3.5
180	180	0	−9.1	*
0	0	180	+2.2	+2.2

trial and used in the calculation. With 78.5 per cent of the third harmonic the secondary loops in the hysteresis curve become more pronounced and cover a range of induction of 14 per cent of *B*. This would increase the measured iron loss, and may well account for the discrepancy of 3.5 per cent.

For $\theta=0°$ the discrepancies between experimental and calculated values are more marked, amounting to three per cent. They are in the usual direction, the experimental loss being less than calculated. I know of no other cause for this than the distortion of the wave due to ohmic drop of potential, already mentioned.

In the experiment with two harmonics the agreement is everywhere within one and one-half per cent; the loss is smaller than expected, with one exception.

In the experiment with seven harmonics the agreement is within one per cent in all cases where the formula can be expected to hold; that is, where $\theta=0°$ for two of the lower harmonics. In the other cases ϕ_1 is not zero and would require a lengthy calculation for its determination. The discrepancy is always on the side of a lower loss than calculated. The loss is always less than with a sine curve unless the third harmonic is introduced in phase with the fundamental, in which case the wave will be flat or dimpled.

TABLE XIII.

m	h	Increase in Loss in Per Cent			
		$\theta = 0°$		$\theta = 180°$	
		Exp.	Calc.	Exp.	Calc.
3	.785	—3.6	—0.6	—22.1	—25.6*
5	.605	—7.1	—3.8	—14.2	—15.0*
5	.39	—1.6	+0.6	—10.1	—10.3*
5	.32	—0.4	+1.3	— 8.6	— 8.8*

It is apparent that transformer losses may be materially reduced by using an appropriate form of wave. When a generator is to be used primarily or principally to supply transformers whose load does not require a sine wave, it would be advantageous to design it with a wave form suitable for the accomplishment of this result. The desired result could be assured by the presence of a considerable component of the third harmonic in reversed phase, and suppressing as far as possible the higher harmonics, unless the phases of these can be controlled. This would give an unmistakably peaked wave.

If the generator be three phase, three wire, it is not desirable to introduce the third or integral multiple of third harmonic. For, if star connected, the harmonic does not appear in the line voltage; if delta connected, a large short-circuit current of the triple frequency may circulate in the armature, wasting power.[7] In this case it would be desirable to choose higher harmonics to produce the desired effect. These can be introduced by a suitable choice of armature teeth, not pushed to a too high flux density. In introducing harmonics, however, it is always necessary to consider the danger from resonance.

If the generator is to be used on a high tension line, the objection must be met that a peaked wave requires a higher maximum emf. for the same effective voltage, and consequently better insulation is required. While this argument carries some weight, it is sometimes overrated. The energy lost by leakage is independent of wave form.

[7] This is discussed by C. P. Steinmetz, Trans. A. I. E. E., **25**, p. 780; 1906: Electrician, **58**, p. 573; 1907.

As for the breakdown of the insulation, it has been shown[8] that the steady voltage necessary for rupture may be greatly increased for a fraction of a cycle without breakdown. Furthermore, most breakdowns on high voltage lines can be traced to surges of abnormal voltage due to some unusual condition, such as opening or closing a switch, not dependent upon the wave form, or to accidental resonance with some high harmonic. The insulation is liable at any time to be subjected to a stress due to twice the effective voltage.[9] It can not be taken for granted, consequently, that a peaked wave would require higher insulation than the same effective voltage in a sine wave. It would be advisable, however, to avoid high harmonics in the wave in order to lessen the danger from resonance effects.

A greater objection to a peaked wave on long transmission lines is the fact that the high maximum emf. requires a larger charging current. How serious this feature of the case may be depends upon the constants of the particular line considered and the amount of power transmitted.

If the secondary have an inductive load, so that its current and the corresponding component of primary current are not in phase with the emf., the difference in wave form due to the ohmic drop in potential will be accentuated, and the effect of the generator emf. upon the iron losses will be less certain.

CONCLUSIONS.

With a given effective electromotive force the iron losses in a transformer depend upon the form factor of the emf, and vary inversely with it.

With a given form of wave, the effect upon the iron losses may be computed approximately from the formulæ derived, providing the higher harmonics are not prominent.

By proper design of the generator supplying transformers, the iron losses may be reduced to a minimum.

WASHINGTON, Oct. 31, 1907.

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[8] C. Kinzbrunner, Electrician, **55**, p. 809; 1905.

[9] P. H. Thomas, A. I. E. E. Trans., **24**, p. 317; 1905.